Analysis of gravitational wave signals from core-collapse supernovae with Non-Harmonic Analysis

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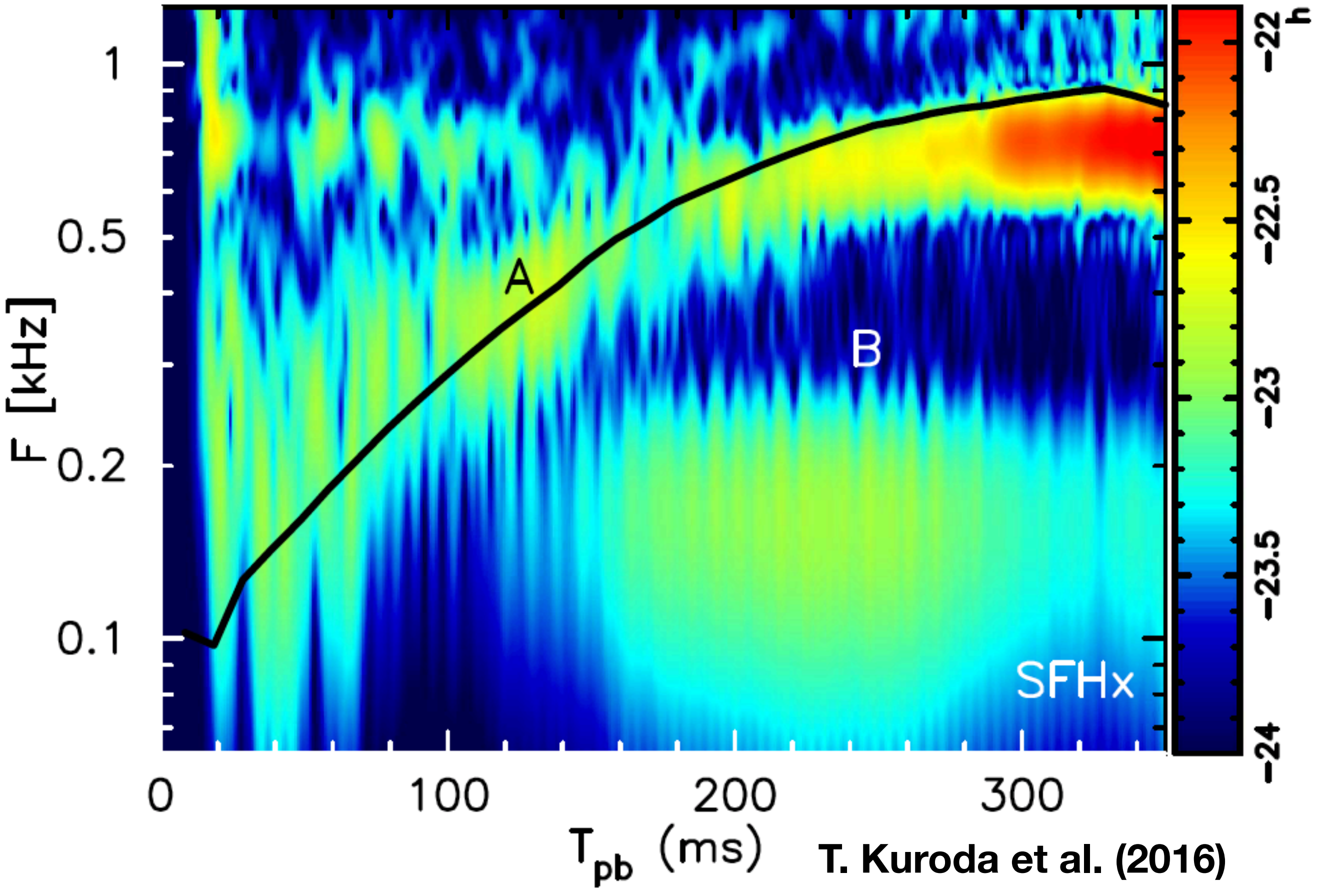
Outline

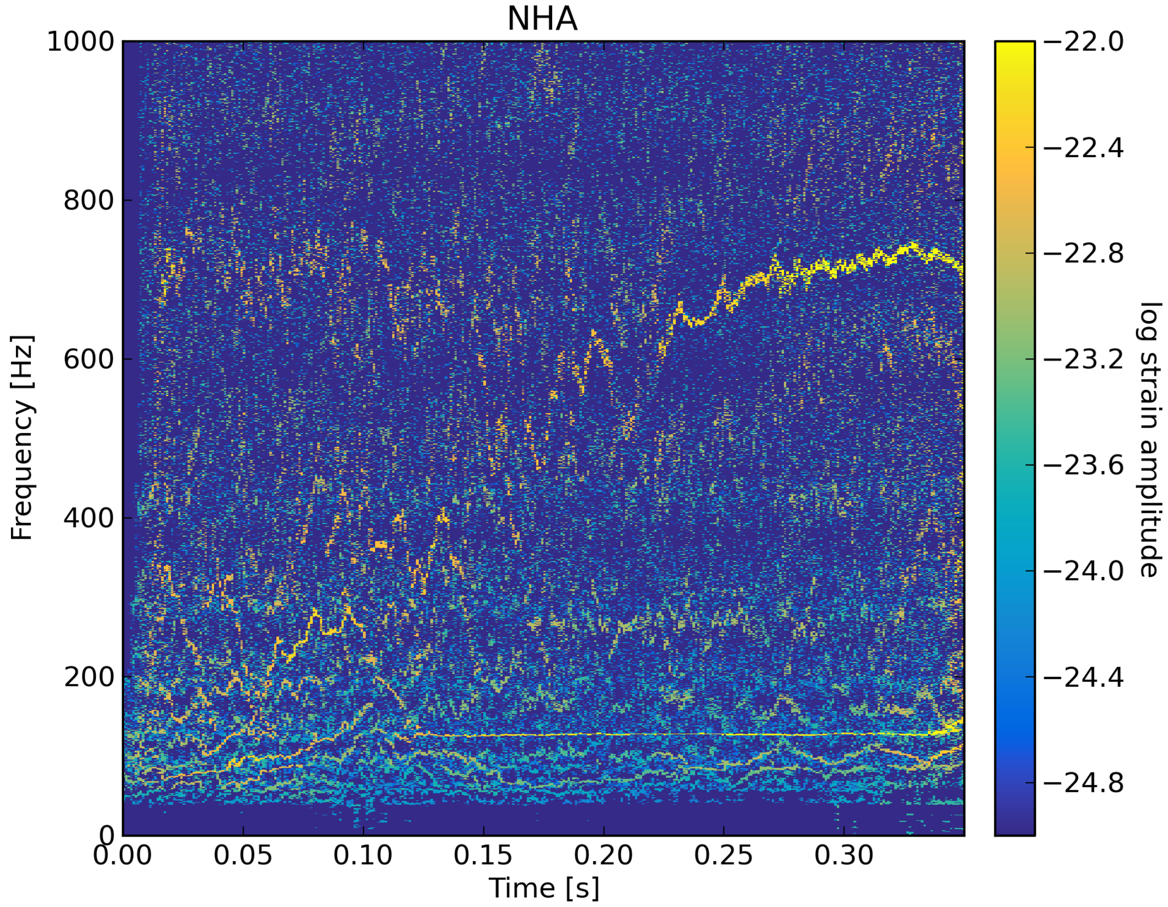
- Introduction to NHA
- Performance of NHA for test signals
- NHA time-frequency map of LIGO events
- NHA time-frequency map of GW signals from CCSN
- Summary

Spectrogram & NHA

Spectrogram or short-time Fourier transform (STFT) is widely used in time-frequency analysis. But one of the weak point in spectrogram is the trade-off between time resolution and frequency resolution. In order to understand the GW modes more accurately, we applied NHA on the GW signals from CCSN.



T. Kuroda et al. (2016)



Non-Harmonic Analysis (NHA)

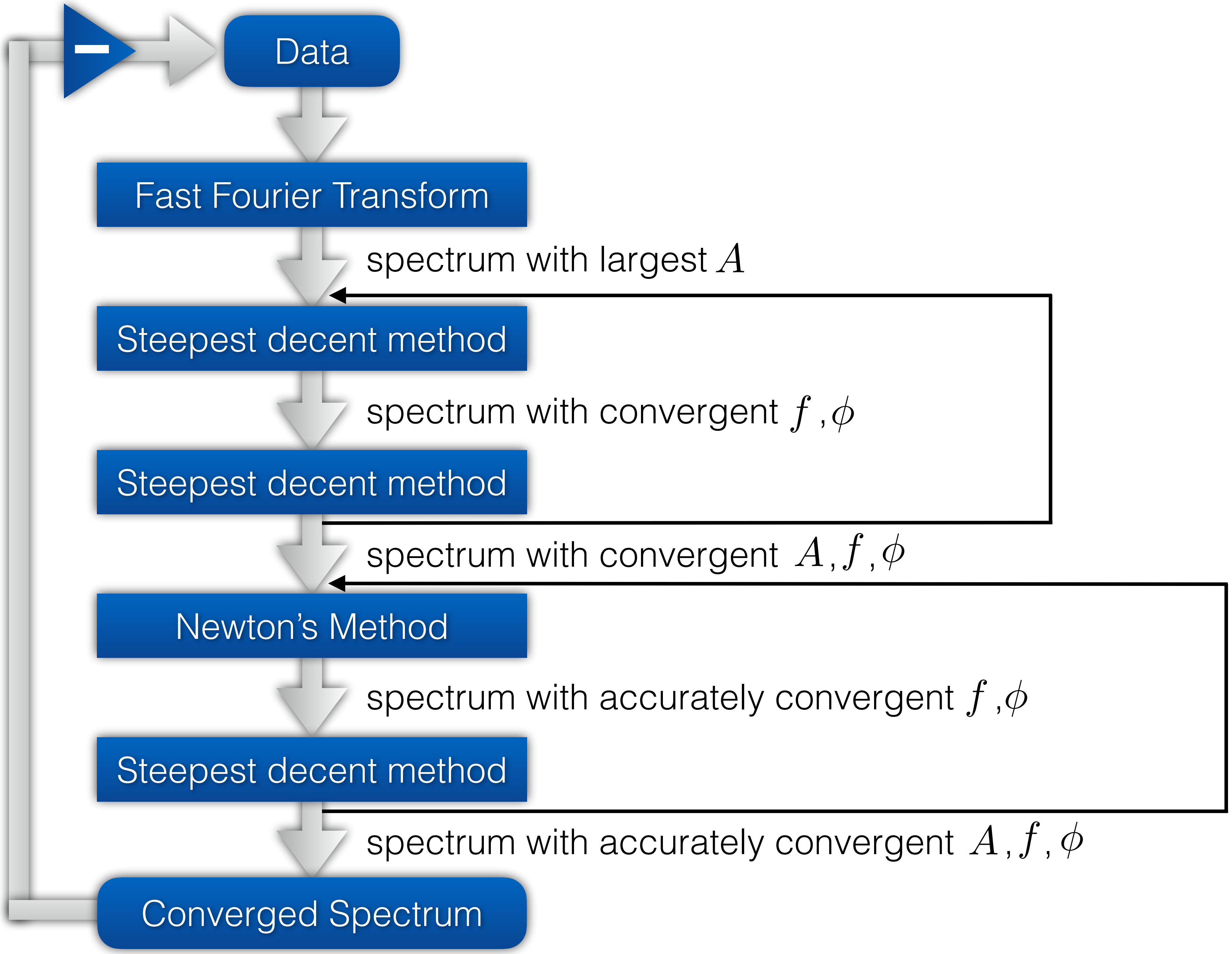
A time-frequency analysis method which has high clarity in both time and frequency resolution.

$$x_j = \sum_{k=1}^{k_{\max}} A_k \cos(2\pi f_k j \Delta t + \phi_k), (j = 0, ..., N-1)$$

For each spectrum, we want to minimize the fitting error.
The cost function is define as

$$F_k(A, f, \phi) = \sum_{j=0}^{N-1} [x_j - A_k \cos(2\pi f_k j \Delta t + \phi_k)]^2$$

NHA algorithm



Data

Fast Fourier Transform

spectrum with largest A

Steepest decent method

spectrum with convergent f, ϕ

Steepest decent method

spectrum with convergent A, f, ϕ

Newton’s Method

spectrum with accurately convergent f, ϕ

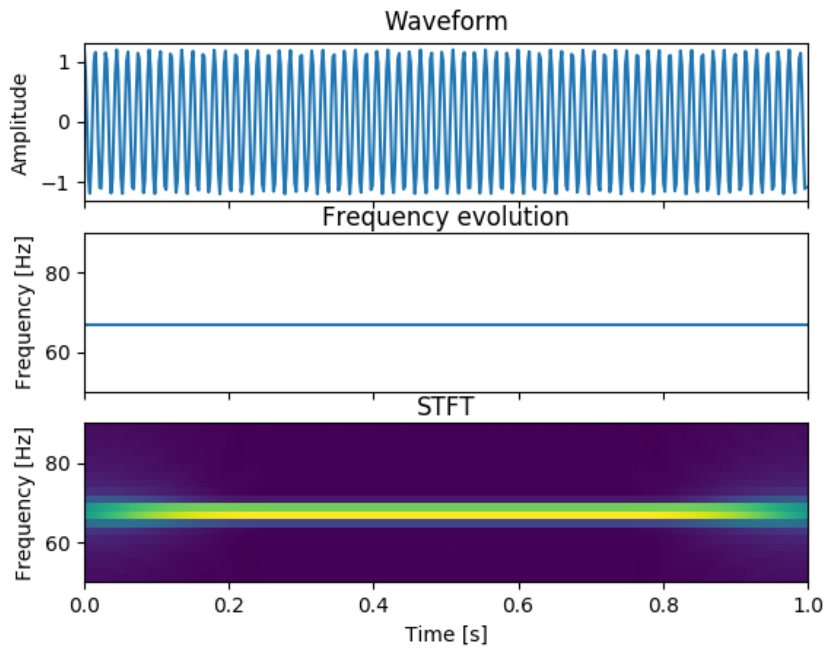
Steepest decent method

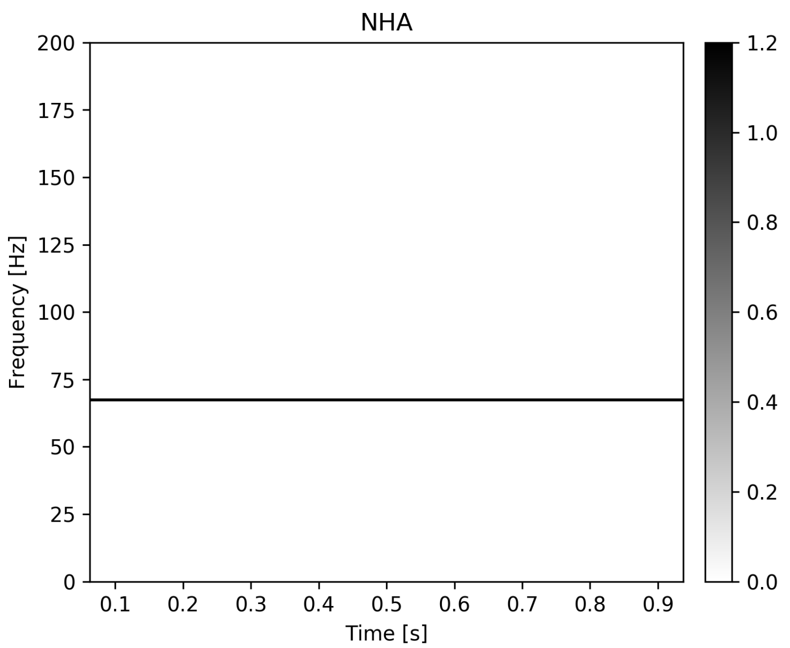
spectrum with accurately convergent A, f, ϕ

Converged Spectrum

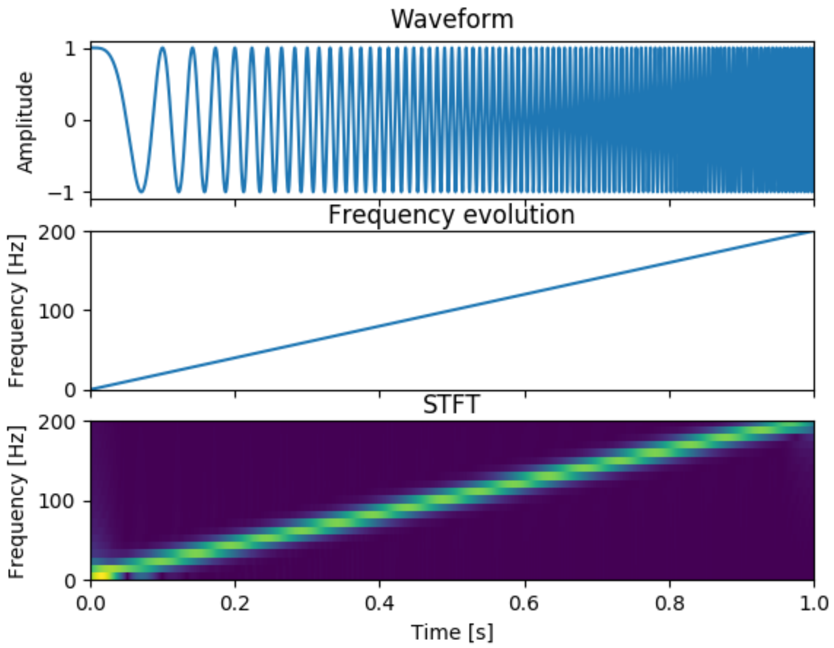
Performance of NHA for test signals

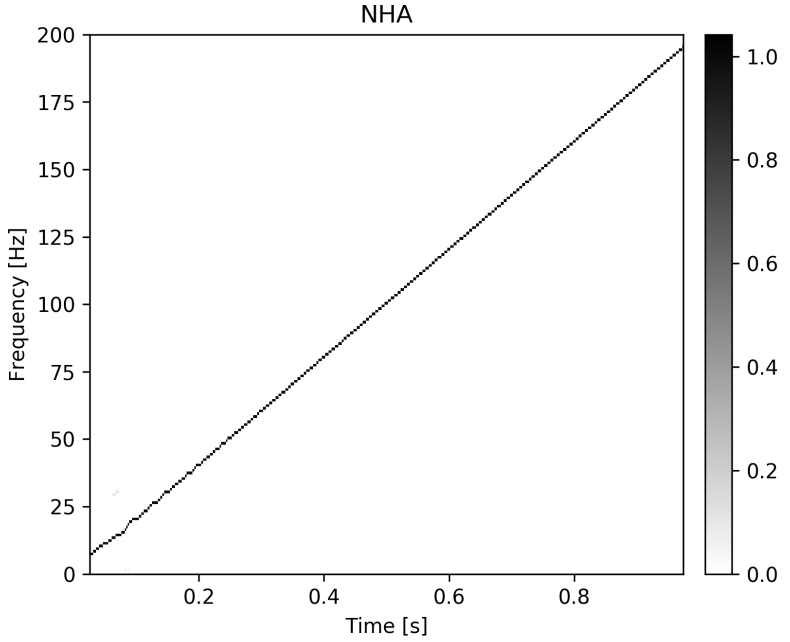
Test signal 1: $x(t) = A\cos(2\pi f t)$ $A = 1.2, f = 66.7\ \mathrm{Hz}$





Test signal 2: $x(t) = \cos(200\pi t^2)$

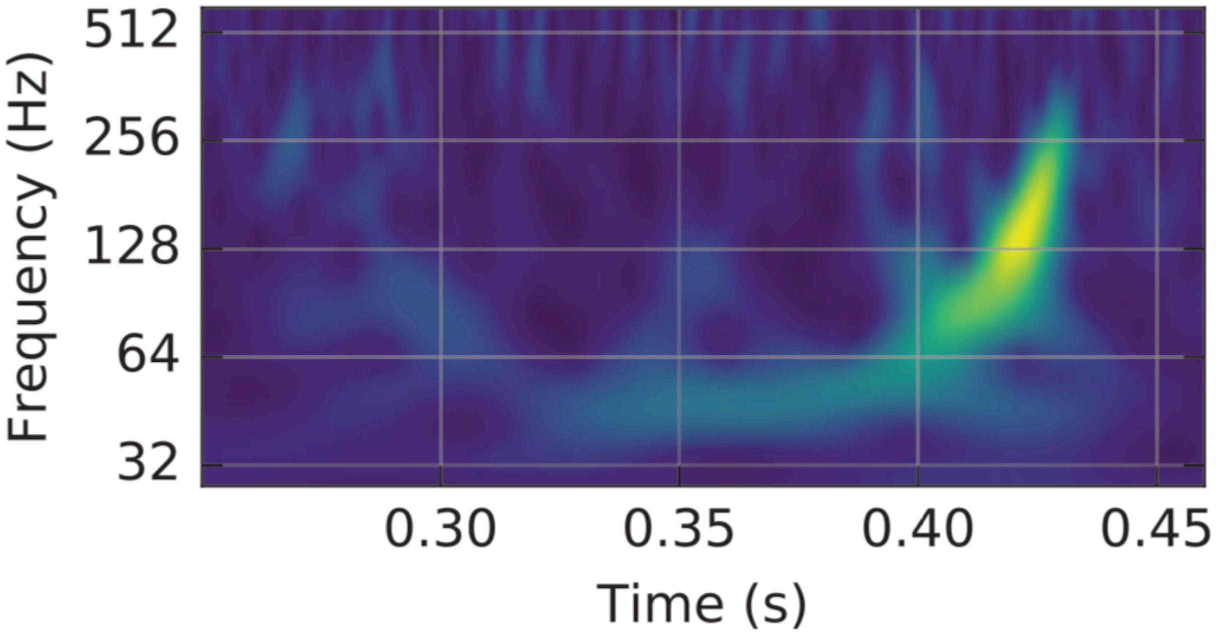


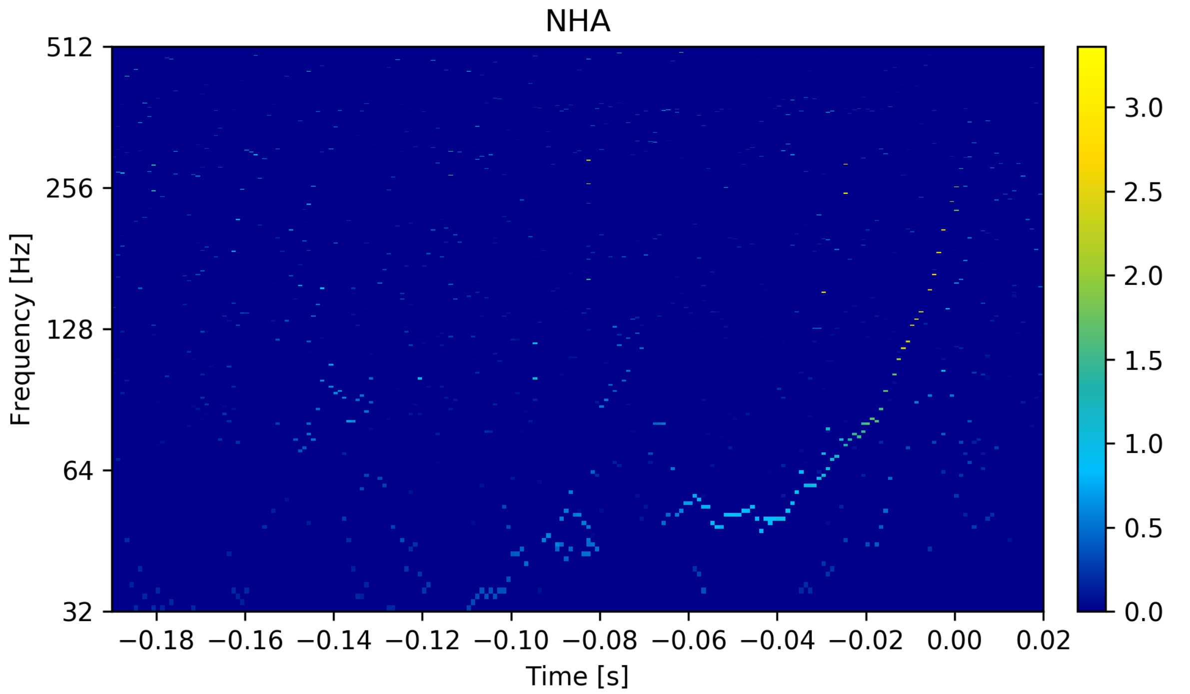


NHA of LIGO events

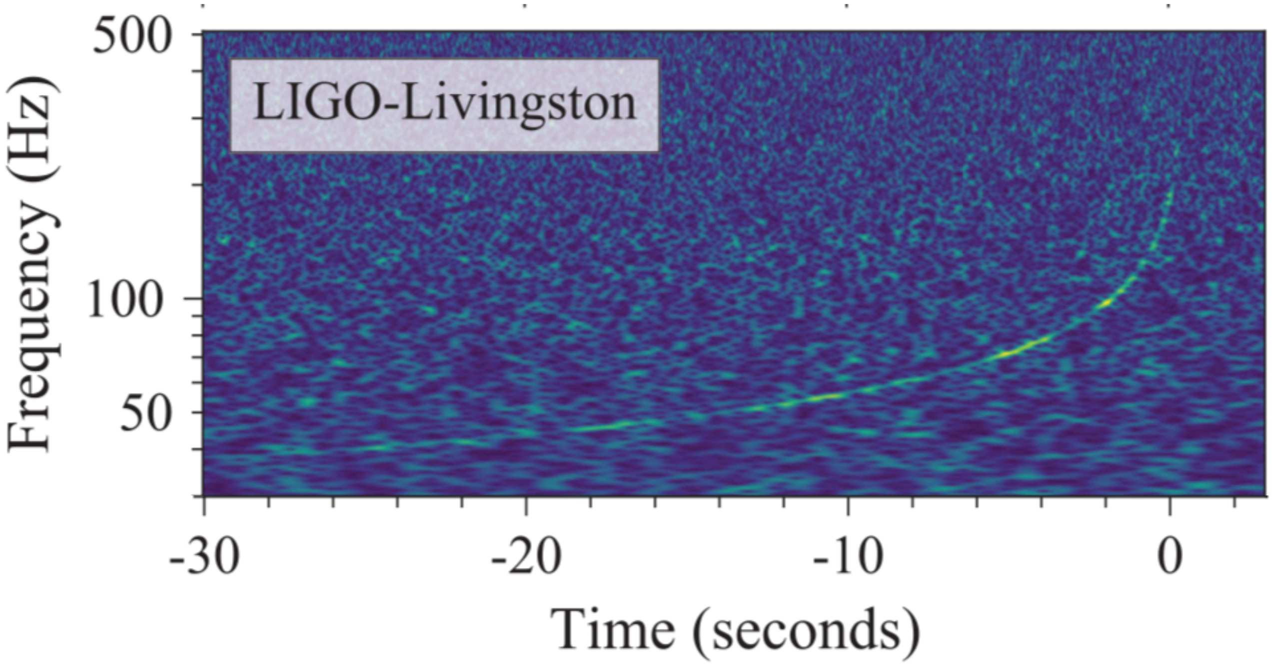
K. Yanagisawa et al. (2019)

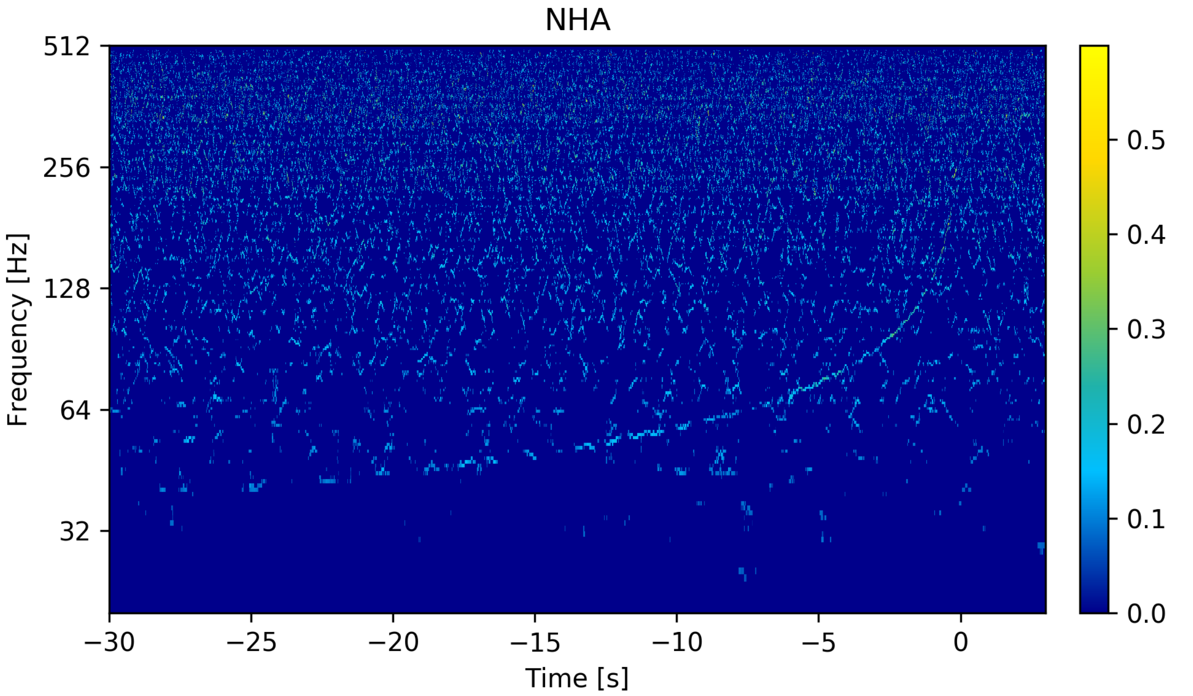
GW150914 LIGO Hanford





GW170817 LIGO Livingston





Simulated GW signals from CCSN

T. Kuroda et al. (2016)

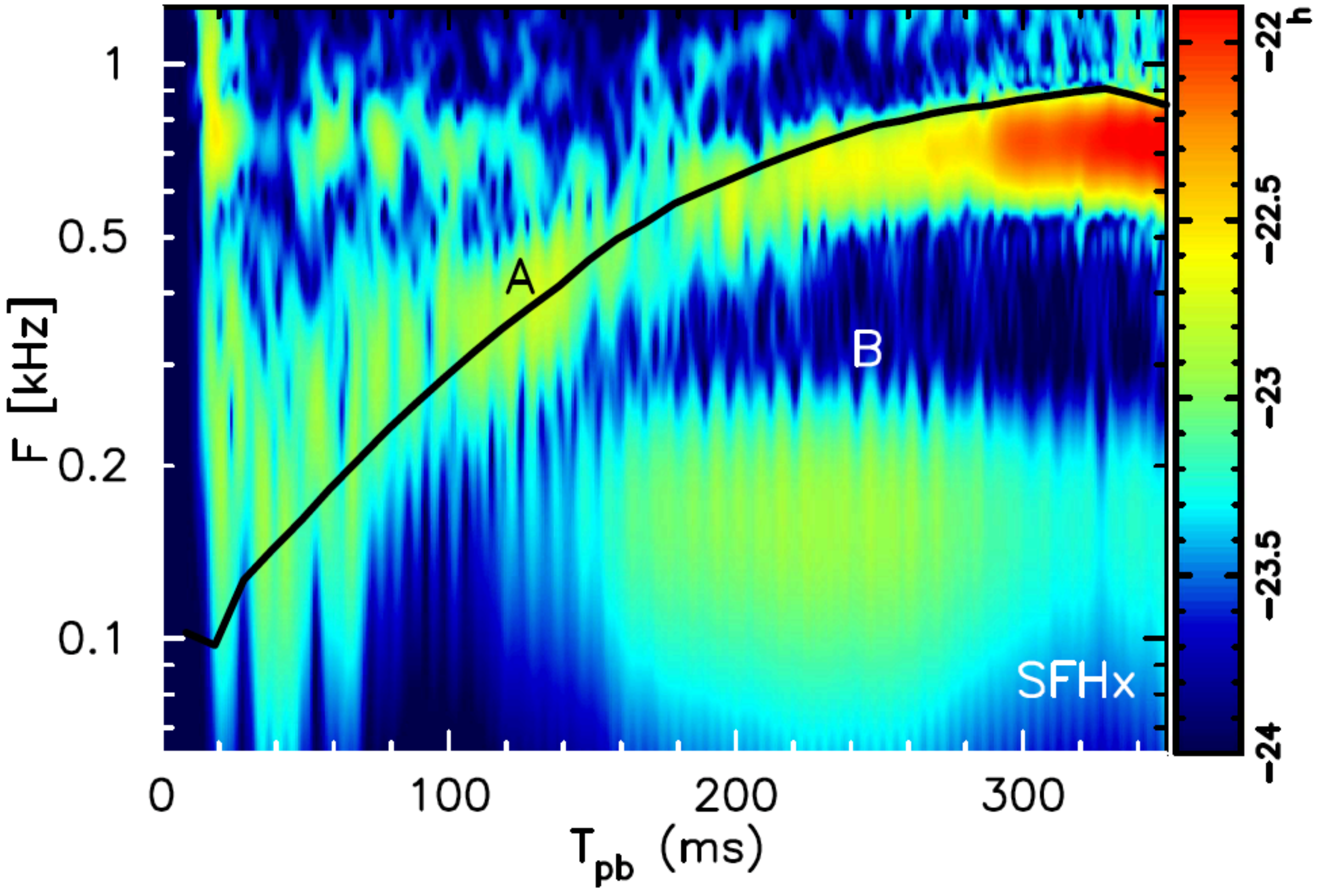
The data from Kuroda et al. are simulated from

- Non-rotating 15 $M_\odot$ star
- Distance 10 kpc
- 3D-GR hydrodynamic simulations
- SFHx model, which is a softer EoS model, shows stronger SASI mode

A: PNS g-mode oscillation

B: SASI activity

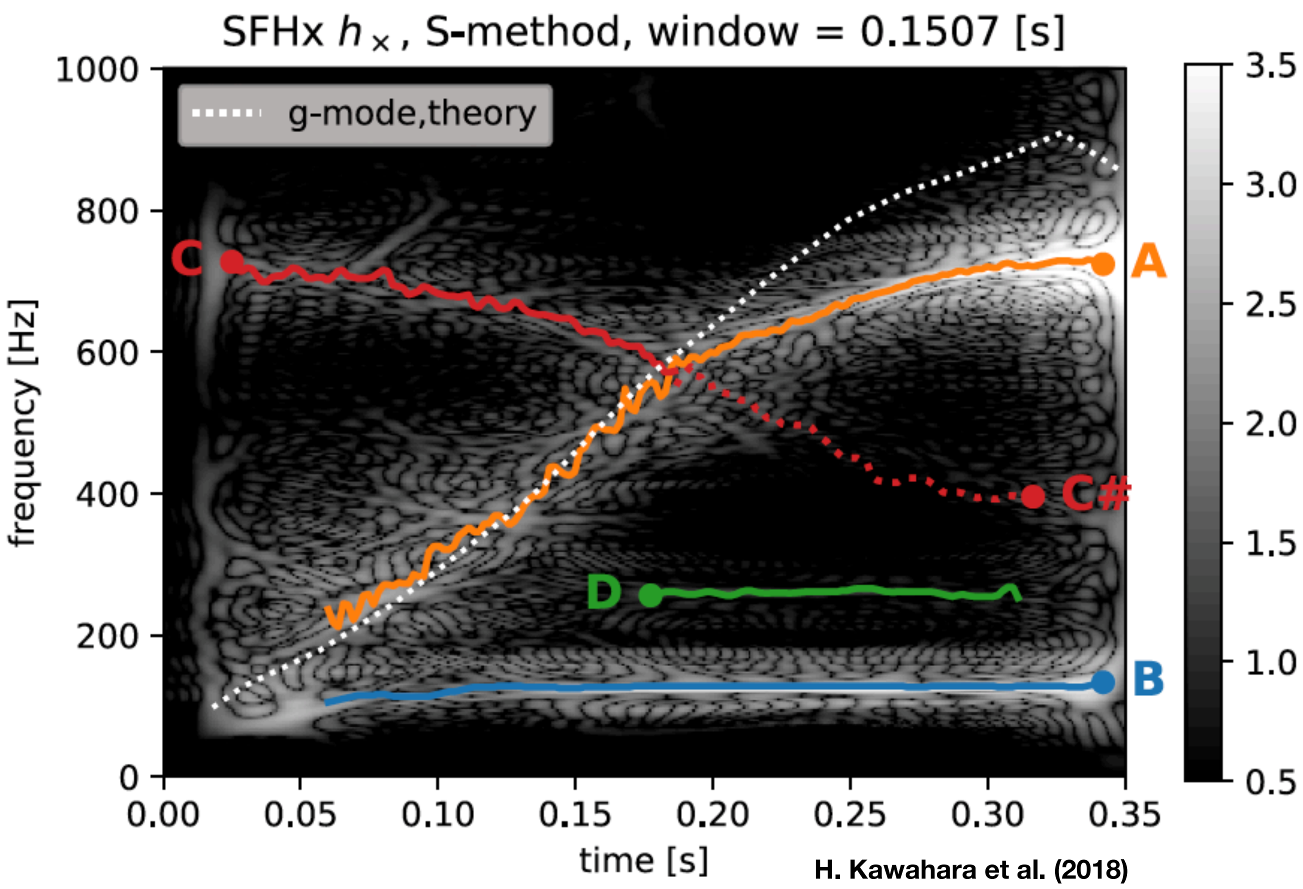
$h_\times$ mode



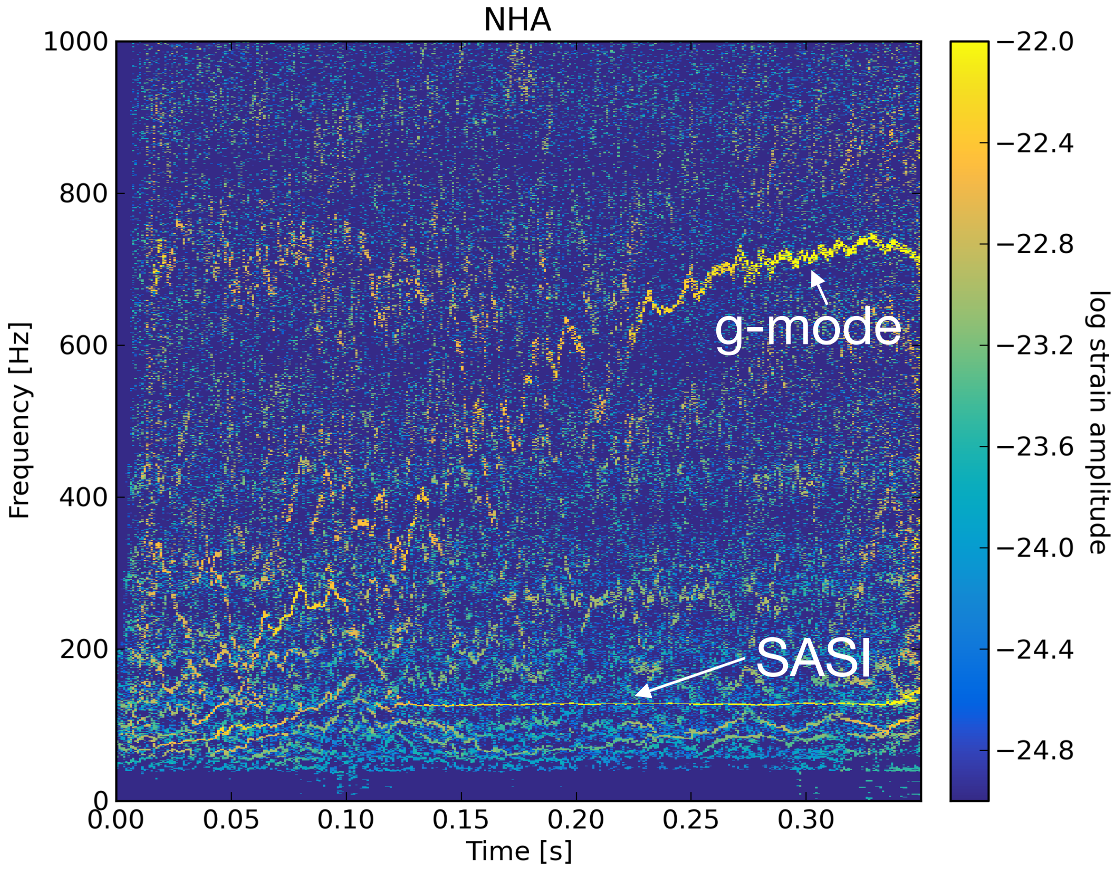
NHA of Simulated GW signals from CCSN

We analyzed a simulated GW signal from CCSN using NHA, and compared the result with spectrogram and Kawahara et al..

- g-mode: we found some oscillations. The oscillations are larger in lower frequency between 300-600 Hz.
- SASI mode: frequency can be resolved very sharply.
- below SASI mode: some possible oscillation mode. We cannot conclude that these are real physical modes.



H. Kawahara et al. (2018)



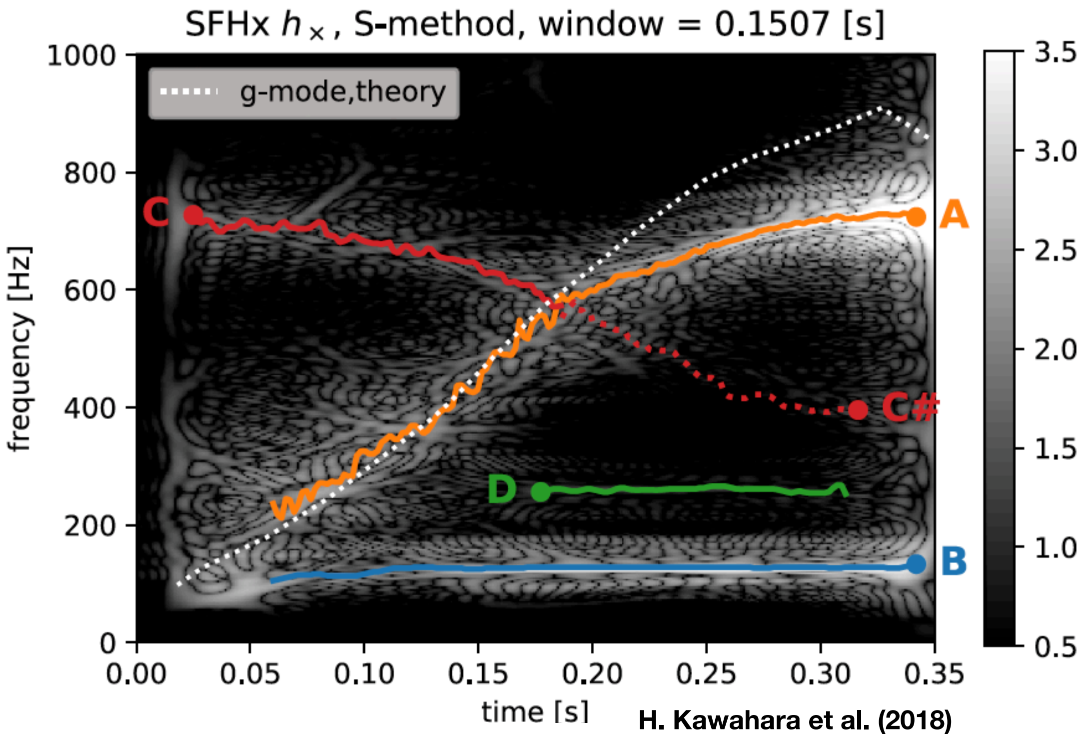
NHA of Simulated GW signals from CCSN

PNS g-mode Kawahara et al. (700Hz)

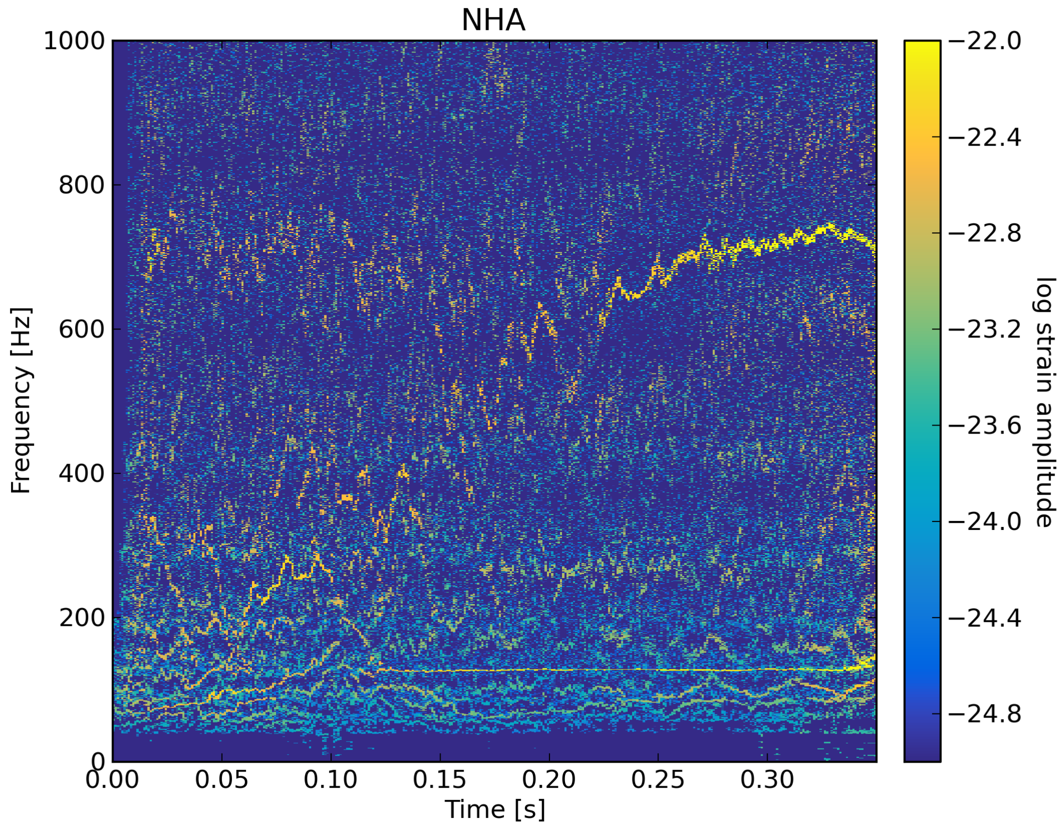
Time	Frequency [Hz]	Amplitude (log)
0.00-0.05	100~200	-22.8~-22.6
0.05-0.10	200~300	-22.5~-22.2
0.10-0.15	300~420	-22.8~-22.2
0.15-0.20	420~630	-22.5~-22.2
0.20-0.25	500~700	-22.5~-22.1
0.25-0.30	660~730	-22.4~-21.6
0.30-0.35	705~750	-21.8~-21.2

SASI mode Kawahara et al. (130Hz)

Time	Frequency [Hz]	Amplitude (log)
0.15-0.20	125~129	-22.2~-22.1
0.20-0.25	127~129	-22.1~-22.0
0.25-0.30	126~129	-22.2~-22.0
0.30-0.35	126~134	-22.2~-21.9



H. Kawahara et al. (2018)



Summary

- NHA is a time-frequency analysis tool which does not have trade-off between time and frequency resolution.

- We performed NHA on test signals and LIGO events, and showed that NHA can show clear GW mode in time-frequency map.

- We analyzed simulated GW signal from CCSN with NHA
    - SASI mode: we obtain precise value of frequency of SASI mode.
    - g- mode: frequency oscillates. The oscillation is larger in lower frequency between 300-600 Hz. This is consistent with the results with S-method (Kawahara et al.)

References

1. K. Yanagisawa, D. Jia, S. Hirobayashi, N. Uchikata, T. Narikawa, K. Ueno, H. Takahashi, H. Tagoshi, PTEP, 2019, 6, 063F01

2. T. Kuroda, K. Kotake, T. Takiwaki, 2016, ApJL, 829, 1, L14

3. H. Kawahara, T. Kuroda, T. Takiwaki, K. Hayama, K. Kotake, 2018 APJ, 867, 2

Backup

GW mode tracking

H. Kawahara et al. (2018)

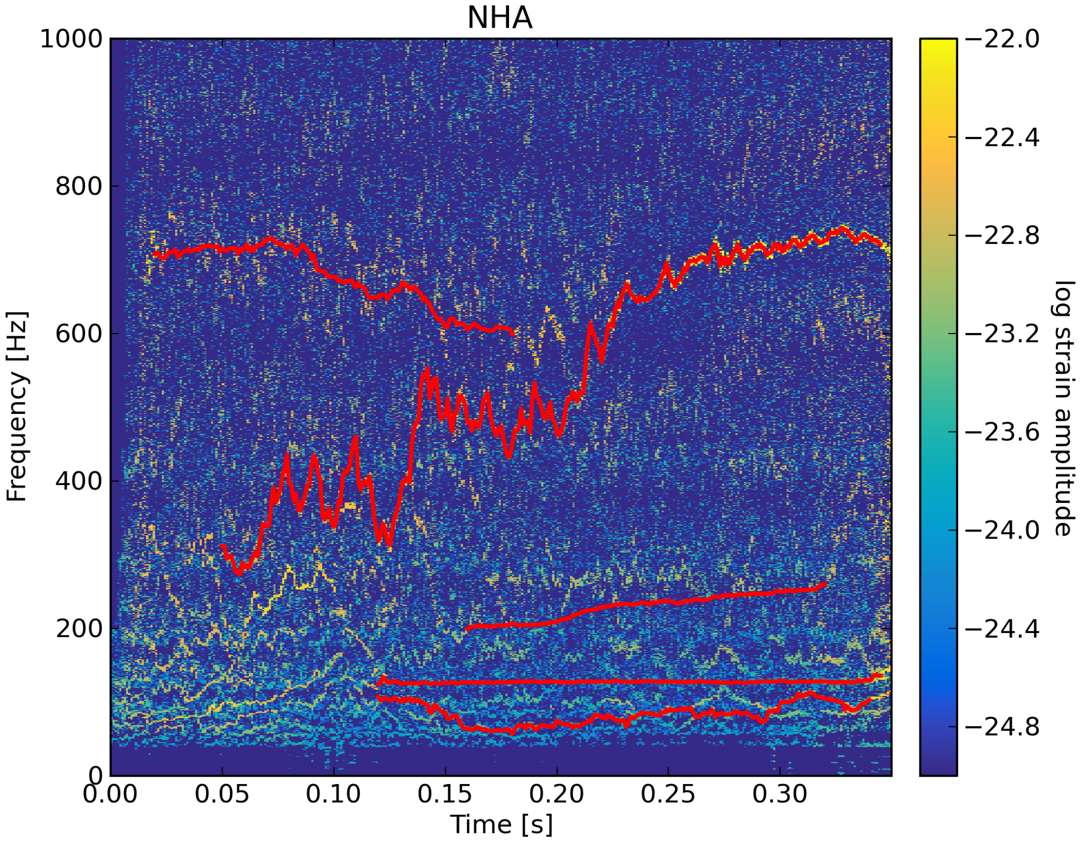
GW modes are tracked by frequency weighted average. The starting points are chosen from the clear peaks in the time-frequency map, and derive the next points by iterating equation.

Iterating equation:

$$f_c^{i+1}(t) = \frac{\int_{f_c^i(t)-w/2}^{f_c^i(t)+w/2} f\, X(f,t)\,df}{\int_{f_c^i(t)-w/2}^{f_c^i(t)+w/2} X(f,t)\,df}$$

Next time step:

$$f_c^0(t_{j+1}) = f_c^{\mathrm{final}}(t_j)$$



S-method

H. Kawahara et al. (2018)

S-method is a modification from Pseudo Wigner Ville distribution, which is not a linear but quadratic transformation.

$$\rho_s(f,t) = 2\int_{-\infty}^{\infty} P(\theta)s(f+\theta/2,t)s^*(f-\theta/2,t)d\theta$$

$$= 2\int_{-\theta_{\rm L}}^{\theta_{\rm L}} s(f+\theta/2,t)s^*(f-\theta/2,t)d\theta$$

$\theta_{\rm L} \rightarrow 0$: Spectrogram

$\theta_{\rm L} \rightarrow \infty$: PWV distribution